Total No of Questions: [08] **SEAT NO. :**

[4456]-101

F.E. (2012)

Engg. Mathematics – I

(2012Pattern)

Time: 2Hours ***Max. Marks : 50***

Instructions to the candidates:

1) ***Attempt Four Questions : Q. No. 1 or 2, Q. No. 3 or 4, Q. No. 5 to 6, Q. No. 7 or 8***
2) ***Figures to the right side indicate full marks.***
3) ***Use of Electronic non-programmable Calculator is allowed.***
4) ***Assume Suitable data if necessary.***

Q1) a) Examine the following system of equations for consistency and solve it, if consistent. [04]

$4x - 2y + 6z = 8,$

$x+y-3z = -1, \quad 15x - 3y + 9z = 21$

b) Examine the following vectors for Linear dependence. [04]

Find the relation between them, if dependent.

(2,-1,3,2), (1,3,4,2) and (3,-5,2,2)

c) If $2 \cos\phi = x + 1/x$, $2\cos\psi = y + 1/y$ [04]

Prove that, $x^p y^q + 1/x^p y^q = 2 \cos(p\phi + q\psi)$

OR

Q2) a) Use De Moivres theorem, to solve the equation [04]

$X^7 + x^4 + I(x^3 + 1) = 0$

b) If $(1+ai)(1+bi) = p + iq$, then prove that, [04]

1. $p \tan[\tan^{-1}a + \tan^{-1}b] = q$

2. $(1+a^2)(1+b^2) = p^2 + q^2$

c) Reduce the following matrix A to its normal form and hence find its rank, where [04]

$$A = \begin{vmatrix} 2 & -3 & 4 & 4 \\ 1 & 1 & 1 & 2 \\ 3 & -2 & 3 & 6 \end{vmatrix}$$

Q3) a) Test convergence of the series (Any One) [04]

1. $\sum_{n=1}^{\infty} \frac{2^n + 1}{3^n + 1}$

2. $\frac{1*2}{3^2*4^2}+\frac{3*4}{5^2*6^2}+\frac{5*6}{7^2*8^2}+......$

b) Expand $40 + 53(x-2) + 19(x-2)^2 + 2(x-2)^3$
In ascending powers of x [04]

c) If $y = x^n \log x$ then, prove that [04]

$$Y_{n+1} = \frac{n!}{x}$$

OR

Q4) a) Solve any one [04]

1) Evaluate $\lim\limits_{x\to\infty} (\cot x)^{\sin x}$

2) Find the values of a and b such that,

$$\lim_{x\to\infty} \frac{a\cos x - a + b x^2}{x^4} = \frac{1}{12}$$

b) Prove that, [04]

$$e^x \tan x = x + x^2 + \frac{5x^3}{6} + \frac{x^4}{2} + ...$$

c) If $Y = \frac{x}{(x+1)^4}$ find Y_n [04]

Q5) Solve any Two of the following [13]

1. Verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ for $u = \tan^{-1}[\frac{y}{x}]$

2. If x = u tan v, y = u sec v

Prove that, $$\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial v}{\partial x}\right)_y = \left(\frac{\partial u}{\partial y}\right)_x \left(\frac{\partial v}{\partial y}\right)_x$$

3. If $u = \frac{x^3 + y^3}{y\sqrt{x}} + \frac{1}{x^7}\sin^{-1}(\frac{x^2+y^2}{2xy})$

Then, find the value of

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$$ At point (1,1)

OR

Q6) Solve any Two of the following [13]

A) If

$$u=(x^2-y^2)f(xy) \text{ then show that}$$

$$u_{xx}+u_{yy}=(x^4-y^4)f''(xy)$$

B) Verify Eulers theorem for homogenous functions

F(x ,y, z) = $3x^2yz+5xy^2z+4z^4$

C) If x = u + v + w, y = uv + uw + vw,
z = uvw
and F is function of x,y,z then prove that,

$$x\frac{\partial F}{\partial x}+2y\frac{\partial F}{\partial y}+3z\frac{\partial F}{\partial z}=u\frac{\partial F}{\partial u}+v\frac{\partial F}{\partial v}+w\frac{\partial F}{\partial w}$$

Q7) a) If $x=v^2+w^2,\ \ y=w^2+u^2,\ \ z=u^2+v^2$ [04]

Find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$

b) Examine for functional dependence for

$u=x+y+z,\quad v=x^2+y^2+z^2,\quad w=x^3+y^3+z^3-3xyz$ [04]

c) Find the extreme values of [05]

$f(x,y)=\ x^3+y^3-3axy,\quad a>0$

OR

Q8) a) If $u^2+xv^2=x+y$ and $v^2+yu^2=x-y$ find $\dfrac{\partial v}{\partial y}$ [04]

b) The resistance R of a circuit was calculated using the formula I = E / R . If there is an error of 0.1Amp in reading I and 0.5 Volts in E, find the corresponding percentage error in R When I = 15 Amp and E= 100 Volts [04]

c) Divide 24 into three parts such that, the continued product of the first, square of the second and cube of the third may be maximum. Use Lagrange's method. [05]